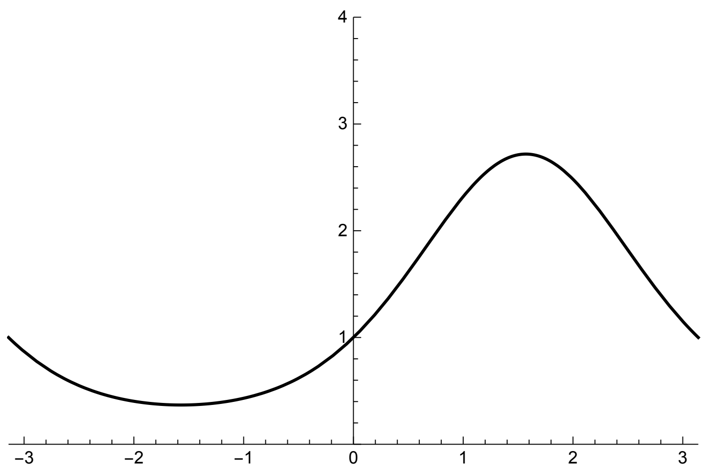
In[28]:=
```
(* The point of this project is to study how the Picard iteration
 converges to the actual solution of a particular ODE. As an example,
we'll take an ODE that we could have solved analytically
 and get some nice polynomial approximations to it;
we'll then plot the approximations and the errors. *)

(* For the example, we'll solve y' = y cos(t) subject to y(0) = 1. The solution
   is y(t) = exp(sin(t)). *)
solution = DSolve[{y'[t] == y[t] Cos[t], y[0] == 1}, y, t]
```

Out[28]= $\{\{y \to \text{Function}[\{t\}, e^{\text{Sin}[t]}]\}\}$

In[29]:=
```
plotSolution = Plot[Evaluate[y[t] /. solution],
  {t, -Pi, Pi}, PlotStyle → {Black}, PlotRange → {{-Pi, Pi}, {0, 4}}]
```

Out[29]=



In[30]:=
```
(* And now for some of the iterates. We'll
 compute and plot these along with the actual solution. *)
approx[t_] = 1;
```

In[31]:=
```
approx1[t_] = 1 + Integrate[approx[s] Cos[s], {s, 0, t}];
```

In[32]:=
```
plot1 = Plot[approx1[t], {t, -Pi, Pi},
   PlotStyle → {Red}, PlotRange → {{-Pi, Pi}, {0, 4}}];
```

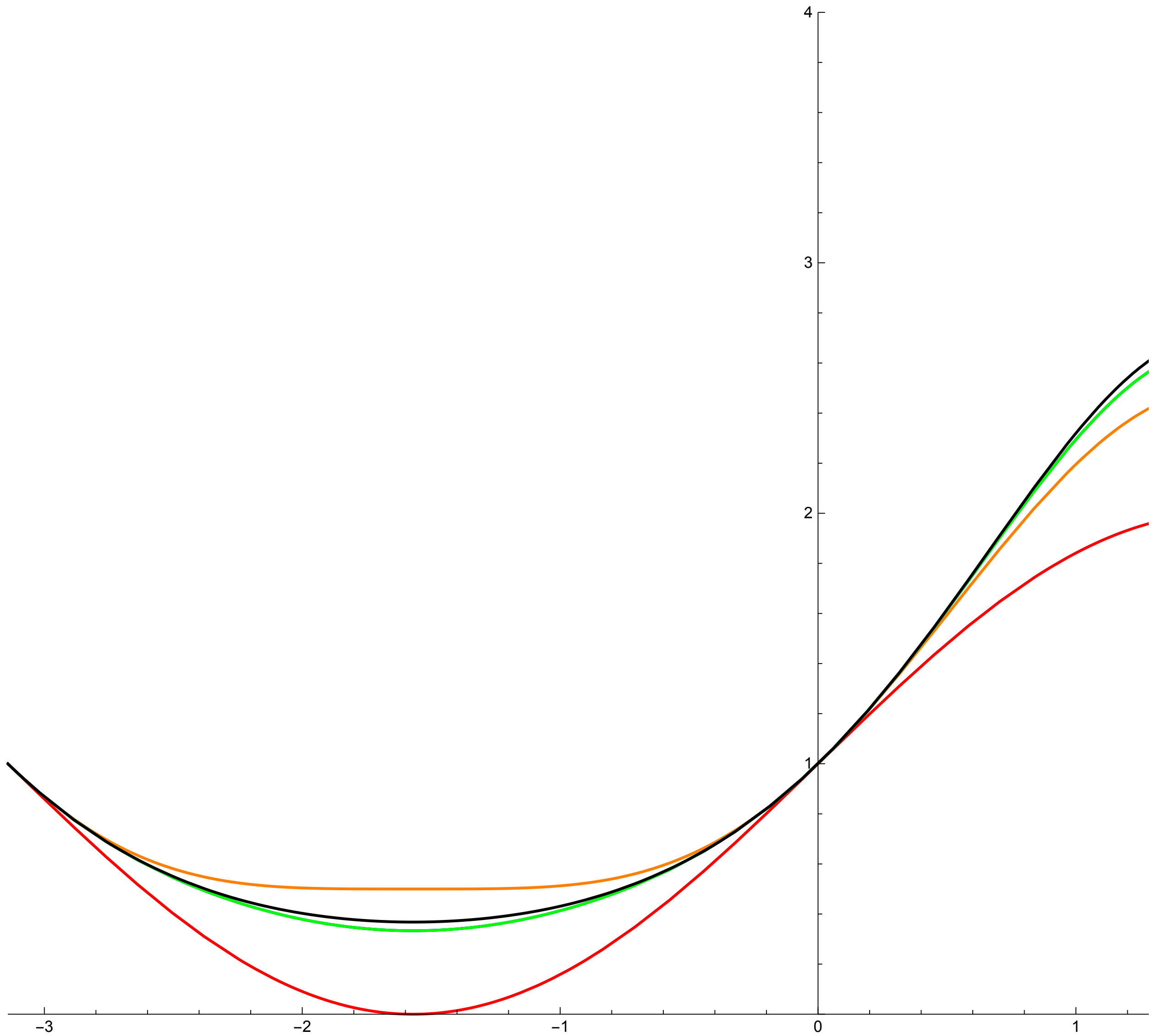
In[33]:=
```
approx2[t_] = 1 + Integrate[approx1[s] Cos[s], {s, 0, t}];
plot2 = Plot[approx2[t], {t, -Pi, Pi},
   PlotStyle → {Orange}, PlotRange → {{-Pi, Pi}, {0, 4}}];
```

In[46]:=
```
approx3[t_] = 1 + Integrate[approx2[s] Cos[s], {s, 0, t}];
plot3 = Plot[approx3[t], {t, -Pi, Pi},
   PlotStyle → {Blue}, PlotRange → {{-Pi, Pi}, {0, 4}}];

approx4[t_] = 1 + Integrate[approx3[s] Cos[s], {s, 0, t}];
plot4 = Plot[approx3[t], {t, -Pi, Pi},
   PlotStyle → {Green}, PlotRange → {{-Pi, Pi}, {0, 4}}];

Show[plot1, plot2, plot3, plot4, plotSolution, PlotRange → {{-Pi, Pi}, {0, 4}}]
```

Out[50]=



In[40]:=

In[41]:=

```
(* So the green curve (approximate) and black curve (exact) are already
 extremely close together after only a few rounds of iteration. The
 blue and green approximation curves are almost indistinguishable. *)
```

In[43]:=

In[44]:=

In[45]:=